Mathematical Modeling
What Are We Missing?

Bruce Grip
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What are students missing when their teachers don't do mathematical modeling in the classroom and don't understand how it differs from "model mathematics." Let's examine authentic mathematical modeling and its potential to deepen conceptual understanding while making the learning of mathematics meaningful.

MaTHink 2017 • CA Math Network Region 10 • February 25, 2017

What are students missing when we don't do mathematical modeling in the classroom?

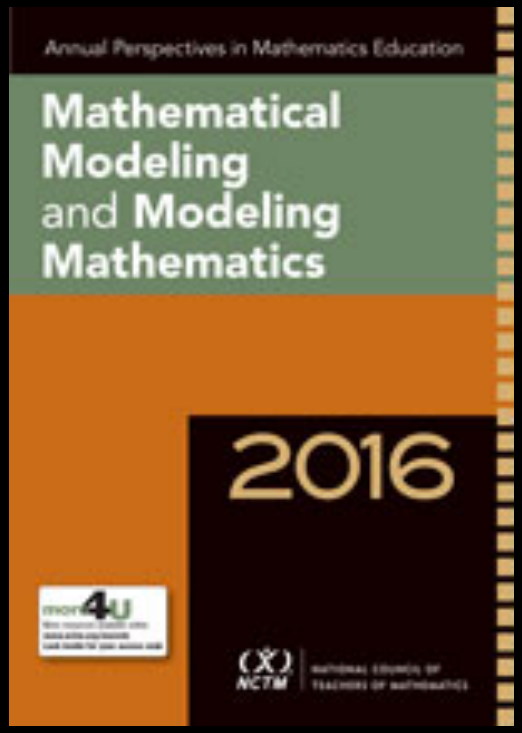
Graham Fletcher Quote

"The lack of mathematical modeling in elementary schools is a cause for concern because it limits our students' ability to reason algebraically and to see the everyday usefulness of mathematics."

Modeling With Mathematics Through Three-Act Tasks, Teaching Children Mathematics, NCTM, April 11, 2016
http://www.nctm.org/Publications/Teaching-Children-Mathematics/Blog/Modeling-with-Mathematics-through-Three-Act-Tasks/

There is no reason to expect that a student who has spent years doing exercises in order to arrive at correct answers will suddenly view mathematics as an empowering tool that helps one make sense of the world. p. 122

Mathematical Modeling and Modeling Mathematics, Annual Perspectives in Mathematics Education, NCTM, 2016.



However, “most mathematics teachers rarely try to link mathematics lessons to the everyday lives of their students who, consequently, don’t expect it” (Steen, Turner, and Burkhardt 2007, p. 287). Students do not always see connections between life and mathematics (Tran and Dougherty 2014) and miss opportunities to apply what they learn to situations around them (Verschaffel, De Corte, and Vierstraete 1999). p. 121

Mathematical Modeling and Modeling Mathematics, Annual Perspectives in Mathematics Education, NCTM, 2016.

Missing the Promise of Mathematical Modeling

Dan Meyer, The Mathematics Teacher, NCTM
Volume 108, No. 8, April 2015, pp. 579-583

I choose to examine modeling for several reasons.

First, speaking strictly personally, I studied mathematics as a child and mathematics education as an adult because of powerful experiences I had using mathematics as a model for the world around me. I want students to have similar experiences.

Second, in my work with teachers in professional development, I find modeling with mathematics (Standard for Mathematical Practice 4, CCSSI 2010, p. 7) to be one of the practice standards most in need of explication. Five different teachers may have five different understandings of its meaning.

Third, mathematical modeling is the standard where mathematics and science meet. The practice standards of the Next Generation Science Standards (NSTA 2012) resemble the mathematical modeling standards of the CCSSM so closely that we should ensure that we get our end right.

Missing the Promise of Mathematical Modeling

Dan Meyer, The Mathematics Teacher, NCTM
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Graham Fletcher Quote

"As elementary school teachers, we have misinterpreted the term model to mean simply the use of manipulatives, a misunderstanding that is causing our students to miss the mark when it comes to modeling with mathematics."

Modeling With Mathematics Through Three-Act Tasks, Teaching Children Mathematics, NCTM, April 11, 2016
http://www.nctm.org/Publications/Teaching-Children-Mathematics/Blog/Modeling-with-Mathematics-through-Three-Act-Tasks/

Missing Out on Math Modeling

Four Questions

What are students missing when we don't do mathematical modeling in the classroom?

What is mathematical modeling, really?

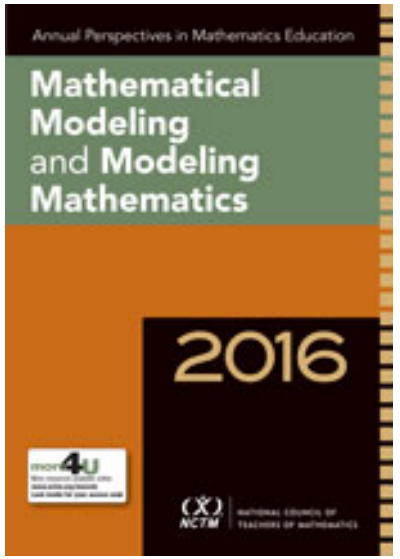
What keeps us from doing mathematical modeling more?

How can we take one step forward to give students more powerful learning experiences that connect the mathematics they learn with the real world in which they live?

Resources for Mathematical Modeling

Mathematical Modeling *and* Modeling Mathematics, Annual Perspectives in Mathematics Education, NCTM, 2016.
https://www.nctm.org/Store/Products/Annual-Perspectives-in-Mathematics-Education-2016--Mathematical-Modeling/

Guidelines for Assessment and Instruction in Mathematical Modeling Education
(GAIMME download at SIAM or COMAP)
http://www.siam.org/reports/gaimme-for_print.pdf



CA Math Framework, Appendix B, Mathematical Modeling
http://www.cde.ca.gov/ci/ma/cf/documents/mathfw-appendixb.pdf

Modeling Across the Curriculum II, SIAM/NSF Report, 2014
http://www.siam.org/reports/ModelingAcross%20Curr_2014.pdf

Mathematical Modeling: Getting Started and Getting Solutions
https://m3challenge.siam.org/resources/modeling-handbook

Missing the Promise of Mathematical Modeling: Dan Meyer
http://www.nctm.org/Publications/Mathematics-Teacher/2015/Vol108/Issue8/?ref=1

Mathematical Modeling Handbook Sampler (COMAP)
http://www.comap.com/product/samples/sampledownloads.html

COMAP = Consortium for Mathematics and Its Applications
SIAM = Society for Industrial and Applied Mathematics

What is mathematical modeling?

What is mathematical modeling according to the CCSS?

- Standards for Mathematical Practice, K-12
- Conceptual Category for High School, 9-12

Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

2 Reason abstractly and quantitatively.

3 Construct viable arguments and critique the reasoning of others.

4 Model with mathematics.

5 Use appropriate tools strategically.

6 Attend to precision.

7 Look for and make use of structure.

8 Look for and express regularity in repeated reasoning.

Standards for Mathematical Practice, CCSS

4. Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems **arising** in everyday life, society, and the workplace.

Mathematically proficient students who can apply what they know, are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Conceptual Categories
Grades 9-12

- Number and Quantity (N)
- Algebra (A)
- Functions (F)
- Geometry (G)
- Modeling (*)
- Statistics and Probability (S)

Conceptual Category: Mathematical Modeling

Modeling links classroom mathematics and statistics to everyday life, work and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data.

…Real-world situations are not organized and labeled for analysis; formulating tractable models, representing such models, and analyzing them is appropriately a creative process. Like every such process, this depends on acquired expertise as well as creativity.

CCSS-Mathematics

Ten Examples

To what extent does each of the ten examples match the CCSS definition of “Model With Mathematics” (SMP#4) or “Mathematical Modeling” (Conceptual Category, 9-12)?

Sort the ten scenarios into three categories according to how well the example matches the CCSS definition:

- Excellent
- Partial
* Poor

Compare Model and Modeling

→ Modeling (verb) uses models (noun).

→Using models does not mean you are doing modeling.*

Compare Model and Modeling

Models serve the math.

Most often confused with mathematical modeling.

Math → Math

Context serves the math.

Application, when done well, provides meaning for the math.
Beware! Meaningless, contrived exercises in disguise!
Note the purpose of the problem or exercise. Why the context?

Math → "Life"

Math serves the context.

Mathematical Modeling

Life → Math → Life

Compare Model and Modeling

Models serve the math.

- What are some models we use in the classroom to help students understand mathematical relationships?

- For example, what models might you use to help students better understand:
  - Fractions
  - Integer operations
  - Solving equations

- *What are some others?*

Compare Model and Modeling

Models serve the math.

- Area model to factor trinomials
- Algebra tiles to solve or simplify
- Colored chips for integer addition and subtraction
- Base 10 blocks
- Cuisenaire rods
- Fraction bars
- Pattern blocks to represent fractions
- Graphs to show intersection of two lines
- Clotheslines as number lines to illustrate relationships
- *What are some others?*

Compare Model and Modeling

Models serve the math. **NOT MATHEMATICAL MODELING**

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(Several textbook bad examples provided in the Ten Examples activity.)

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Math serves the context. **MATHEMATICAL MODELING**

- Where and when should you place security personnel?
- How much is Kevin Durant worth to his team?
- Is global warming real? Should changes be made?
- Should I drive farther to fill up with less expensive gas?
- When should I retire?
- What is the fastest route for UPS or Fedex delivery?
- Should I start my own business?
- Earthquake prediction.

What is NOT mathematical modeling?

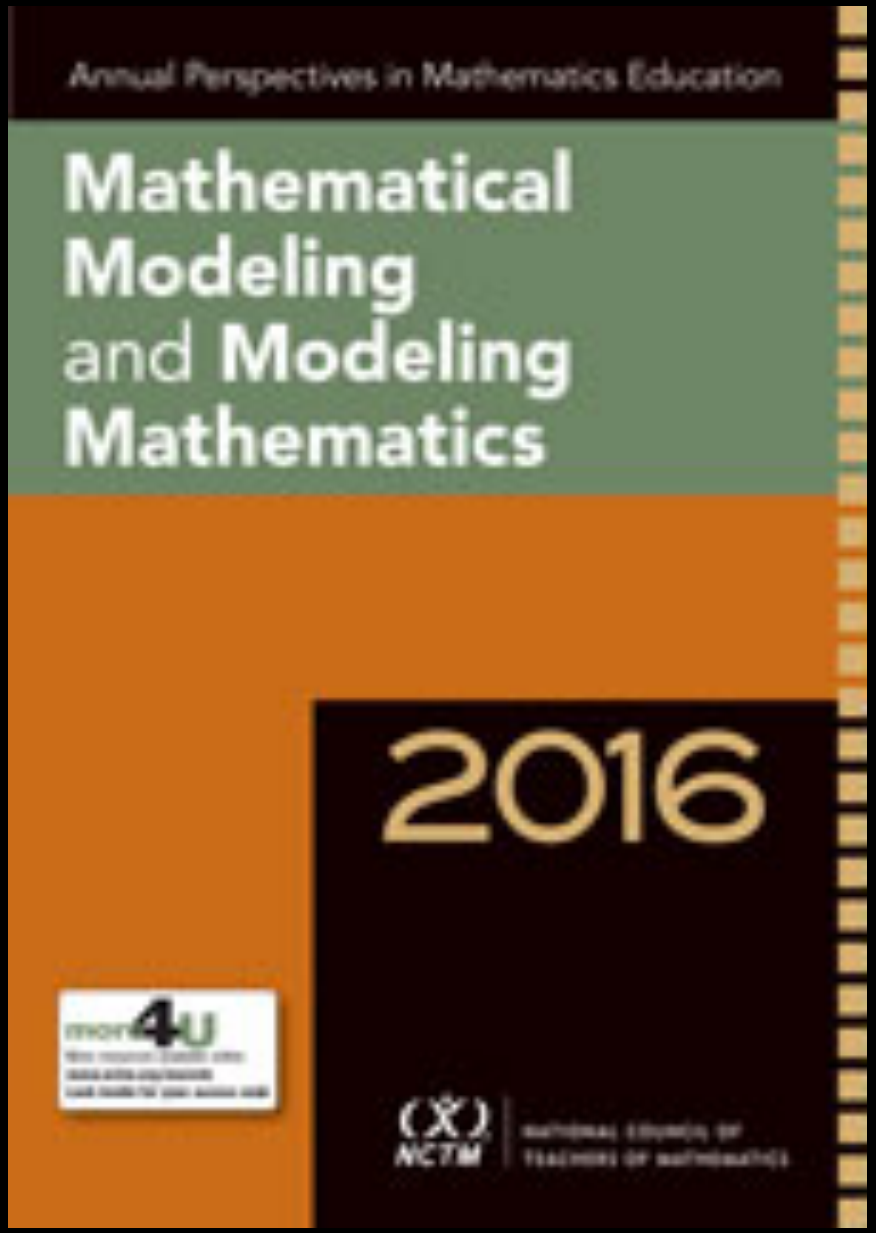
- It is NOT showing students how to do a procedure.
- It is NOT the use of manipulatives.
- It is NOT modeling just because the textbook says so.
- It is NOT modeling because the word “model” appears.
- It is MORE THAN using mathematical models such as tables, graphs, equations, drawings, etc.

What is NOT mathematical modeling?

- It is not just starting with a real-world situation and solving a math problem; it is returning to the real world situation and using the mathematics to inform our understanding of the world (i.e. contextualizing and de-contextualizing, see MP.2)

- It is not beginning with the mathematics and then moving to the real world; it is starting with the real world (concrete) and representing it with mathematics.

California Mathematics Framework, Appendix B, p. 1.

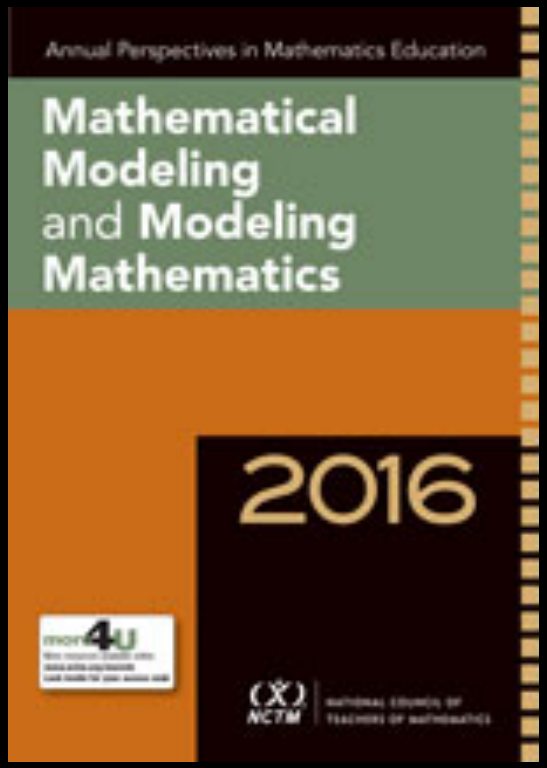


How is Mathematical Modeling different from Modeling Mathematics?

Mathematical modeling and *modeling mathematics* are not the same, and so it is unfortunate that the same root word of "model" appears in two distinct constructs.

The distinction between models of mathematics and mathematical modeling is not always clear in US standards documents and in the mathematics education literature.
More specifically, the Common Core State Standards for Mathematics (CCSSM) uses the terms *model* and *modeling* to mean both *modeling mathematics* and *mathematical modeling* without clarifying the difference in meaning….

NCTM, 2016, p. 3.



G.A.I.M.M.E.

GUIDELINES FOR ASSESSMENT AND INSTRUCTION IN MATHEMATICAL MODELING EDUCATION

- *Consortium for Mathematics and Its Applications (COMAP)*
- *Society for Industrial and Applied Mathematics (SIAM)*

1. Identify the problem.
2. Make assumptions and identify variables.
3. Do the math.
4. Analyze and assess the solution.
5. Iterate.
6. Implement the model.

p. 12, GAIMME Report

Mathematical Modeling Process

1) **Begin with a real-world situation.**
   Problem to solve • Situation to understand • Decision to make
   Product or process to create or improve • Question to answer • "I wonder…"
   May want to re-visit and change

2) **Identify key factors and simplify.**
   Variables • Assumptions • Features to ignore
   May want to re-visit and change.

3) **Create and solve mathematical models.**
   Pictures • Drawings • Graphs • Equations • Tables • Spreadsheets • Maps
   Apply and/or learn relevant mathematics (in context!)

4) **Analyze the result.**
   Will it work? • Is it doable? • Simplify? • Add back complexity?
   Change the question? • Change assumptions? • Change the model?

5) **Report and support your results.**
   Get feedback • Accept suggestions • Act

Begin with a real-world situation.

Problem to solve • Situation to understand • Decision to make
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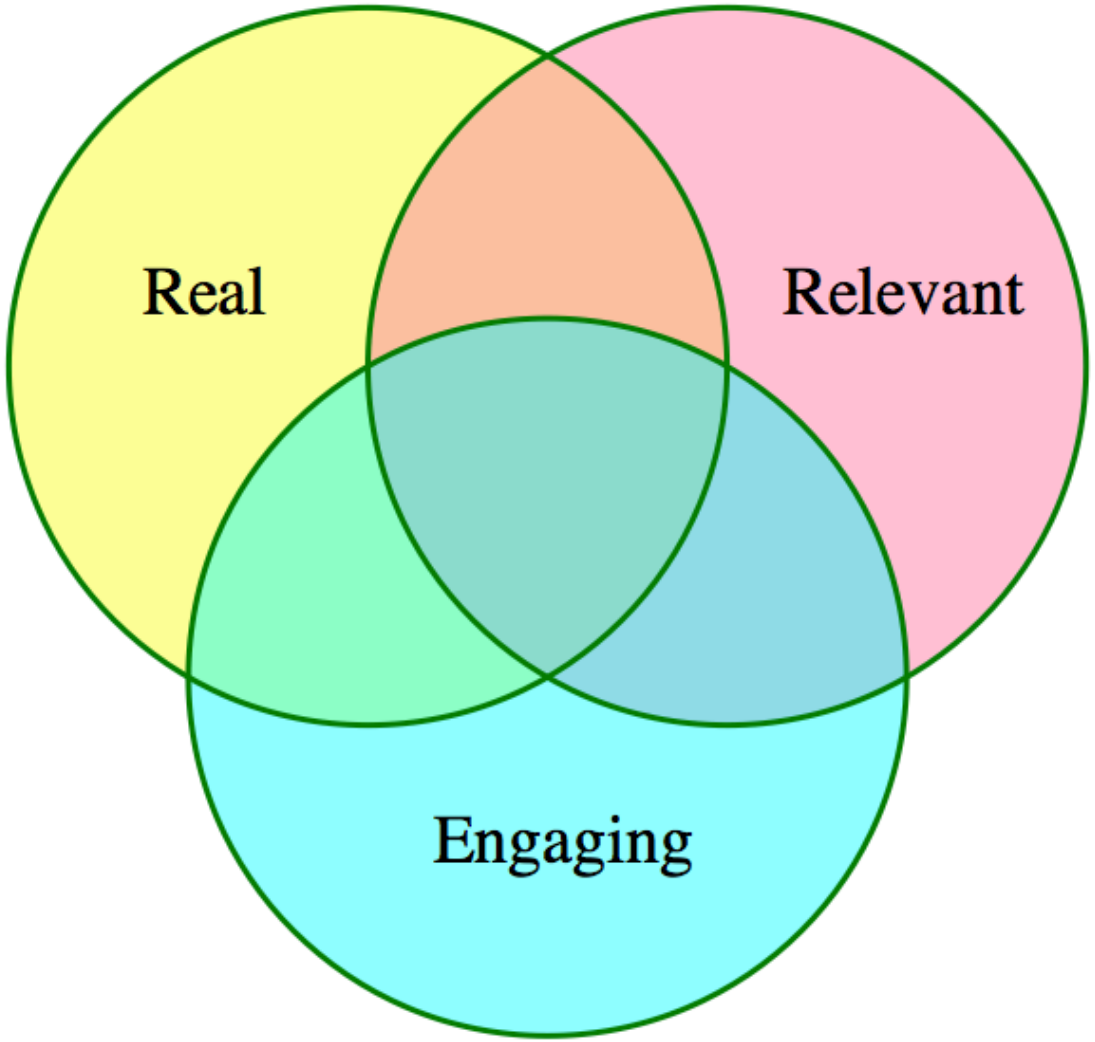
What real-world situation would be relevant and engaging for your students?

GAIMME: Identify the problem.

Engaging • Relevant • Real



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Some of my favorite contexts:

Lemonade Stand
Farmer's Market Snow Cones
Hot Dog Cart
Heather's Pies
Fundraisers
Amusement Park
School Event (Dance, Banquet)
Rehearsal Dinner (my son)
Wedding Venue and Catering
Movie Ratings
Yearbook Sales

Pizza Choices
Soda Can Packaging
Drug Dosing
Fitness Club Membership
Phone and Data Plans
Twitter Growth Data
Starbuck's Growth Data
Mauna Loa CO Levels Data
Global Warming
Retirement
Automobile Purchase or Lease

GAIMME: Identify the problem.

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Real-world context + Non-mathematical purpose

What do you wonder?

What questions should be considered?

GAIMME: Identify the problem.

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Bruce Grip, 2013

GAIMME: Make assumptions and identify variables. .

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GAIMME: Do the math.

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GAIMME: Analyze and assess the solution. Iterate as needed.

GAIMME: Implement the model.

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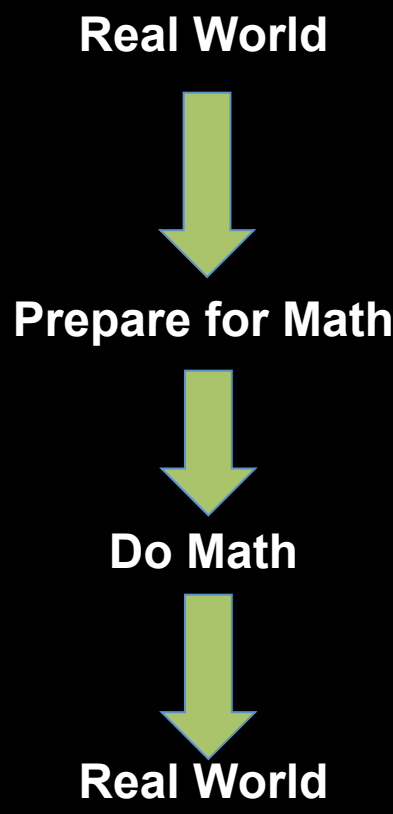

Bruce Grip, 2013

Modeling In a Nutshell

Start With Life

- Problem to solve, question to answer, decision to make, real-life situation to understand.
- Life can be complex → Simplify!

Do The Math!

- Use available tools, models, representations.
- Learn new ones (and the requisite mathematics)
- Attend to accuracy, appropriateness, relevance

Check Back With Life

- Apply, revise, extend.
- Cycle through the process as many times as needed.

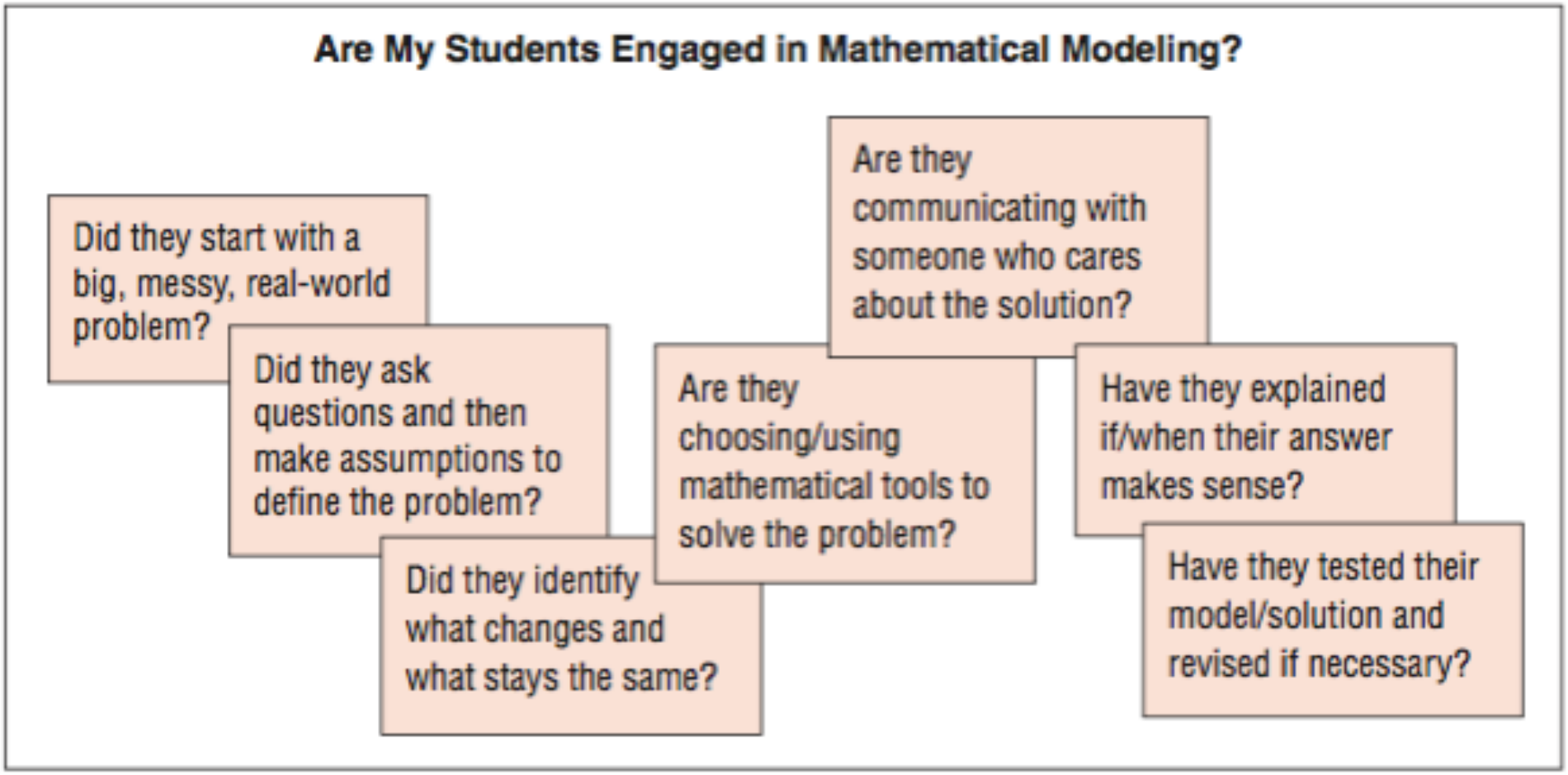

Fig. 23.2. Assessing students' engagement in mathematical modeling. Adapted with permission from Rachel Levy, IMMERSION program

Mathematical Modeling and Modeling Mathematics, NCTM, 2016, p. 256

Compare Model and Modeling

Models serve the math.	**NOT MODELING**
Context serves the math.	**NOT MODELING**
Math serves the context.	**MATH MODELING**

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Mathematical Modeling
Life → Math → Life

What are students missing when we don't do mathematical modeling in the classroom?

The Potential of Mathematical Modeling

"…mathematical modeling can also support the learning of mathematics in terms of motivation, comprehension and retention, and in terms of what mathematics is and how it can be used."

Blum and Borromeo Ferri (2009)
NCTM, p. 6.

Graham Fletcher Quote

"The lack of mathematical modeling in elementary schools is a cause for concern because it limits our students' ability to reason algebraically and to see the everyday usefulness of mathematics."

Modeling With Mathematics Through Three-Act Tasks, , NCTM, April 11, 2016

The Potential of Mathematical Modeling

Are your students experiencing the power of mathematical modeling?

1. Add **meaning** to school mathematics.
   Learn mathematics to answer real-world questions.
   "When am I ever going to use this?"

2. Rich, real-world problems can provide **access** to students with different mathematical backgrounds and experience.

3. Utilize many of the **Standards for Mathematical Practice** and not just SMP#4. (Make sense, persevere, contextualize, de-contextualize, reason, support, justify, communicate, use tools appropriately, attend to precision,…)

4. Build **conceptual understanding** upon concrete real-world experiences and contexts. (*Function, Slope, Ratios, Systems, Area, Volume,…*)

Where do I find real life problems for modeling?

Resources

Dan Meyer
http://blog.mrmeyer.com/
https://docs.google.com/spreadsheet/ccc?key=0AjlqyKM9d7ZYdEhtR3BJMmdBWnM2YWxWYVM1UWowTEE#gid=0
www.101qs.com

Yummymath
http://www.yummymath.com/birds-eye-of-activities/

Mathalicious (paid subscription)
http://www.mathalicious.com/lessons

Robert Kaplinsky
http://robertkaplinsky.com/
https://docs.google.com/spreadsheet/ccc?key=0AtbjNWC9XVmudGV4VUQ1OFRpMmNNUU5peIZMVkJrWkE#gid=0

Andrew Stadel, Estimation 180
http://www.estimation180.com/

Resources

Inside Mathematics
insidemathematics.org

La Cucina Matematica (John Stevens and Matt Vaudrey)
http://www.classroomchef.com/links/

Illustrative Mathematics
https://www.illustrativemathematics.org/

Emergent Math
http://emergentmath.com/my-problem-based-curriculum-maps/

MEA Library, University of Nevada Las Vegas
http://wordpress.unlvcoe.net/wordpress/

CPALMS
Search: Model Eliciting Activity (Math)
http://www.cpalms.org/public/search/Search

MARS Tasks
http://map.mathshell.org/tasks.php
http://map.mathshell.org.uk/materials/tasks.php

Mathematical Modeling Contest Problems
http://www.comap.com/undergraduate/contests/mcm/previous-contests.php
http://www.math.cornell.edu/~mcm/
https://m3challenge.siam.org/?_ga=1.151531133.594450183.1465424726

LEMA Modeling Tasks (download)
http://www.lema-project.org/web.lemaproject/web/eu/tout.php?v3=/web.lemaproject/web/eu/page_resources.htm

NCTM Illuminations
http://illuminations.nctm.org/LessonsList.aspx?grade=all&standard=all

Society for Industrial and Applied Mathematics (SIAM)
https://www.siam.org/publicawareness/competitions.php

Society for Industrial and Applied Mathematics (SIAM)
http://m3challenge.siam.org/about/mm/?

NCTM Illuminations
http://illuminations.nctm.org/LessonsList.aspx?grade=all&standard=all

Building Better Modeling

DRAFT PROPOSAL

+ Real —

Authentic — Contrived

+ Relevant —

Relevant — Irrelevant

+ Engaging —

Captivating — Boring

+ Support —

Open/Independent — Guided

+ Whole Modeling Process —

Invites Full Process + Iteration — Partial Process

+ Complexity • Accessibility —

Complex • Messy • May Require Background Knowledge/Advanced Tools/Modeling Experience

Overly-simplified • Accessible

Level of Support

Do your students generate questions? Pursue curiosity? Make decisions about which model fits? Check for reasonableness? Justify conclusions?

Hand Holding
No decisions to make

Teacher Facilitates as needed
Students use Math Practices

"Be less helpful." – Dan Meyer
"When students seem to need help, seem to help them." – Henry Pollak

Conclusion

Suggestions for moving forward at all grade levels.

Why are teachers ***not*** doing mathematical modeling in their classrooms?

- Not enough time
- Content standards focus
- Teachers don't know how (not how they learned math)
- Lack resources (textbooks inadequate)
- Available problems are stupid, contrived, irrelevant,…
- Belief students must learn necessary skills first
- We <u>are</u> doing it (or we think we are)

Early and Middle Grades Modeling
Five Guiding Principles

The Early and Middle Grades section of the GAIMME report presents five guiding principles to help teachers facilitate this practice as they support students' growth as modelers and monitor the students' acquisition of mathematical tools:

- Modeling (like real life) is open-ended and messy.
- When students are modeling, they must be making genuine choices.
- Modeling problems can be developed from familiar tasks.
- Assessment should focus on the process and not on the product or pieces only.
- Modeling happens in teams.

Mathematical Modeling and Modeling Mathematics, NCTM, 2016, p. 257

SUGGESTIONS FROM THE MATHEMATICAL MODELING TASK FORCE, 2011.

Student success implementing the modeling process in high school mathematics, college majors and careers should be enhanced if younger students:

- Begin to ask their own life questions and see mathematical processes and skills as an important part of understanding and solving these problems.
  - How do you share a quantity with a group of students?
  - What is the shortest path?
  - What is the best way to package items of similar or different shape?
  - How many of one item will fit inside another?
  - How much will it cost? How long will it take? How fast must you travel?
  - Who will arrive first?
  - How do you create a scale model of an object?
  - How do you make a profit when selling a particular product?
  - How do you make a stronger building, bridge or tower?
  - Which pizza is the best deal? Which size of product is the best value?
  - How is math used in crime scene investigations?
  - What is the best text, phone, or pay-per-view plan?
  - Which container holds the most of a quantity?

Continued….

SUGGESTIONS FROM THE MATHEMATICAL MODELING TASK FORCE, 2011.

Student success implementing the modeling process in high school mathematics, college majors and careers should be enhanced if younger students:

- Experience the use of different ways to model and understand a problem. Experience with manipulatives, graphs, measuring, equations, technology, etc. Students should want to use more than one approach to solving a problem. Students should know that some problems have a best or most efficient solution and for other problems, "it depends."

- Provide students with experience asking questions in non-threatening ways to "help me understand your thinking" rather than "prove you're right."

- Provide examples of real people in real careers using mathematical modeling to solve real problems that utilize mathematics familiar (or mostly familiar) to the students at the particular grade level.

- Explore some rich math problems that span several lessons or days. Possibly keep track of learning through a chart with "What I Know" and "What I Need to Know."

Mathematical Modeling Notes

Whimsical problems, contrived problems, pure math problems can also be valuable, engaging and fun.

The process of mathematical modeling may be applied and practiced using whimsical or contrived problems.

Sometimes exploration of pure mathematics leads to real-life applications.

Mathematical Modeling Notes

The development and practice of important skills is more meaningful when students see the skills are relevant to solving real-world problems. Skill development does not have to precede problem solving but can be woven into the process of solving problems using mathematical modeling.

Mathematical Modeling Notes

We all do mathematical modeling every day. Learning the process of mathematical modeling and applying it in the classroom will take time. Do not rush it. Do not ignore it. Collaborate with colleagues (and your students) and embrace a way of learning mathematics that connects the math to the real world.